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TRANSITION FROM LAMINAR TO TURBULENT FLOW AT SUBSONIC AND SUPERSONIC SPEEDS

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(Paper presented at Conference on High-Speed Aeronautics, Polytechnic Institute of Brooklyn, January 20, 1955.)

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List of Symbols

c	velocity of propagation of breakdown of laminar flow
c_r	wave velocity of sinusoidal wave in laminar boundary flow
d	diameter of tube
k	height of roughness element
l	reference dimension of body
L	scale of turbulence
L_o	distance from leading edge to origin of disturbance
L_w	wave length of disturbance
M	Mach number
Re_o	Reynolds number at transition for smooth plate when discussing data on rough plate
Re_t	Reynolds number at transition $\frac{U_o x_t}{\nu}$
T_i	recovery temperature, absolute
T_o	adiabatic stagnation temperature, absolute
T_s	surface temperature, absolute
u	local velocity at point x, y in boundary layer
u_1	speed of leading edge of turbulent spot
u_k	local velocity in boundary layer at point, $x = x_k$, $y = k$
$\overline{u}$	average velocity of flow in a tube
U_o	velocity of free stream
u', v', w'	components of turbulent velocity fluctuations
x	distance along surface from leading edge

x_k	distance of single roughness element from leading edge
x_t	distance of beginning of transition from leading edge
y	distance normal to surface
α	2π/wave length for sinusoidal wave in laminar boundary layer
β_r	2π times frequency for sinusoidal wave in laminar boundary layer
δ	boundary layer thickness from 4-term Pohlhausen polynomial approximation
δ^*	displacement thickness of boundary layer, i.e. $\int_0^\infty (1 - u/U_0)\, dy$ for incompressible flow, $\int_0^\infty (1 - \rho u/\rho_0 U_0)\, dy$ for compressible flow
δ_k^*	displacement thickness of boundary layer at $x = x_k$
λ	Pohlhausen pressure gradient parameter $(\delta^2/\nu)(d\, U_0/d\, x)$
ν	kinematic viscosity of fluid, free-stream conditions
θ	half angle of wedge of turbulence at source
ρ	local fluid density
ρ_0	free-stream fluid density

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Introduction

While the early cave-dweller may have often observed the phenomenon of transition from laminar to turbulent flow in the rising filaments of smoke from his outdoor campfire on a still day, scientific observation and study of the problem began with Osborne Reynolds in the 1880's. [1] He observed in flow through a long cylindrical tube the change in the pressure-flow relationship from the theoretical viscous flow prediction, the sudden breakdown of a filament of dye and the ensuing rapid diffusion of the dye throughout the tube. Transition occurred at a constant value of the Reynolds number $\frac{\bar{u}d}{\nu}$ of 2300 where $\bar{u}$ was the average velocity of the fluid, d the diameter of the tube, and ν the kinematic viscosity of the fluid. It was found later that the Reynolds number for transition depended on the amount of initial disturbance present at the tube entrance. With great care to reduce disturbances, transition Reynolds numbers of 20,000 or more have been attained.

When the skin friction on a plate had been computed by Blasius on the basis of the boundary layer concept formulated by Prandtl in 1904, it was suspected that transition to turbulent flow also occurred in boundary layer flow. The experiments of Froude [2] which had been reported many years before indicated that the skin friction on a plate varied as the 1.85 power of the speed whereas the boundary layer theory predicted a variation as the 1.5 power. Further indirect evidence resulted from studies of the drag of spheres between 1911 and 1914 by Eiffel, Föppl, and Wieselsberger which led to the discovery of the marked decrease in drag coefficient of spheres in a critical range of Reynolds numbers and the dependence of this range on wind tunnel turbulence. In 1914 Prandtl [3] explained the critical Reynolds number range as due to transition of the flow in the boundary layer. The resulting turbulent flow by promoting more vigorous mixing delays separation of the flow from the surface and decreases the pressure drag of the sphere.

Direct evidence of transition in the boundary layer came from a detailed study of the velocity distribution in a boundary layer of a thin sharp-edged plate parallel to the flow made by B. G. van der Hegge Zijnen, [4] a student of Burgers, some 20 years after Prandtl's original boundary layer paper. These measurements clearly showed departures from the theoretical results of Blasius beginning at an x - Reynolds number $\frac{U_0 x}{\nu}$ of 300,000. Typical turbulent flow distributions were found at x - Reynolds numbers greater than 500,000. Later measurements were made by Hansen and Elias at Aachen and by the author and his colleagues at the National Bureau of Standards [5]. The boundary layer along a smooth thin flat plate parallel to the flow in a stream of uniform velocity and hence without longitudinal pressure gradient is the simplest possible case for study both theoretically and experimentally. Much effort has been devoted to clarifying our understanding of the birth of turbulence in this so deceptively simple experimental situation.

The present paper undertakes to review the more illuminating experimental results on transition at low subsonic speeds, particularly those for a flat plate, and the related theoretical investigations. The available information will then be synthesized into a somewhat speculative picture of the transition process in incompressible flow. This will be followed by a summary of the much more limited data for supersonic flow.

Part I. Incompressible Boundary Layer Flow

Factors Affecting Transition in Incompressible Flow

In general terms the occurrence of transition in a two-dimensional boundary layer has been found to be dependent on the free stream turbulence, the pressure gradient, the curvature of the surface, the roughness of the surface, and the temperature of the surface relative to the free-stream temperature. Values of the x-Reynolds number for transition ranging from less than 100 thousand to about 14 million have been observed.

In quantitative studies the location of transition depends to some extent on the method of definition and measurement. A common method depnds on the observed variation of mean speed as measured by a pitot tube or hot-wire anemometer traversed along the surface at a suitably chosen distance from the surface. The velocity decreases slowly as a result of the skin friction, but increases rapidly when turbulence is generated. The location of transition is commonly defined as the point of minimum velocity, although strictly speaking this is but the beginning of the transition region. Other methods are based on the characteristic features of oscillograms of velocity fluctuations indicated by a hot-wire anemometer either observed directly or heard by passing the output through earphones, on listening to pressure fluctuations with a stethoscope, on changes in heat transfer or rate of evaporation with change from laminar to turbulent flow, etc. The results are reasonably concordant and the differences are small compared to the large effects of many of the controlling factors. In this paper the transition Reynolds numbers quoted refer to the beginning of the transition region and to free-stream density and viscosity.

Effect of Free Stream Turbulence

Extensive studies have been made of the effect of free-stream turbulence on transition in the boundary layer of a smooth flat plate without pressure gradient by Dryden, [5] Schubauer and Skramstad, [6] and Hall and Hislop [7] . The results are shown in Fig. 1 with an additional isolated observation by Wright and Bailey [8] . The free-stream turbulence is defined as the quantity $\dfrac{100\sqrt{1/3\,(u^{1^2} + v^{1^2} + w^{1^2})}}{U_o}$ where u^1, v^1, and w^1 are the root mean square values of the turbulent velocity fluctuations and U_o is the free-stream velocity. The scale of the free-stream turbulence may also have some effect but it is not evident in the available experimental data. The transition location is that determined from pitot tube traverses. The transition Reynolds number Re_t varies from 90 thousand to 2.8 million.

Effect of Pressure Gradient

When there is a variation of pressure along a flat plate produced for example by placing the plate in a converging or diverging flow, the location of transition is changed. The x - Reynolds number is increased when the pressure is falling in the downstream direction (converging flow) and decreased when the pressure rises downstream. Systematic experiments for the case of uniform pressure gradients along the plate varied over a wide range are not available. Wright and Bailey [8] made some measurements at a turbulence level of about 0.2 percent for which Re_t was 2 million with uniform pressure. Relatively small favorable pressure gradients increased Re_t to as much as 2.5 million whereas small adverse gradients decreased Re_t to 0.7 million. Liepmann [9] made measurements on the convex side of a curved plate, expressing his results in terms of the local Reynolds number based on momentum thickness, which is approximately proportional to the square root of the x-Reynolds number. The equivalent Re_t of 2 million for zero gradient was increased to about 2.9 million for small favorable gradients and decreased to 0.8 million for adverse gradients.

In the experiments described, the pressure gradients were small, the Pohlhausen parameter λ being only of the order of 1. Much larger gradients are found on airfoils. For comparison with the flat plate results some allowance must be made for the effects of the pressure variation in modifying the boundary layer thickness at a given distance from the leading edge. This may be done approximately by defining an equivalent flat plate Re_t as equal to 1/3 $Re_{\delta *}^2$ where $Re_{\delta *}$ is the Reynolds number based on the displacement thickness of the boundary layer. The highest values obtained in favorable pressure gradients on extremely smooth surfaces both in wind tunnels of low turbulence and in flight are approximately 14 million. (This value should not be confused with values of the airfoil Reynolds number at which drag rise occurs, based on wing chord.)

When the adverse pressure gradient is sufficiently large and extends over a sufficient distance, the free-stream flow separates from the surface and the wall boundary layer may continue as a laminar free shear layer in the fluid. However transition is observed to occur in such shear layers at a much lower Reynolds number (based on the thickness of the layer) than in boundary layers on the wall. When transition occurs, the resulting turbulence may cause the flow to reattach to the surface leaving a separation bubble and continuing as a turbulent boundary layer. Thus transition may occur on airfoils as a result of separation of the laminar layer. [10]

Effect of Curvature of the Surface

A systematic study of curvature effects was made by Liepmann. [9] On a convex surface for values of the radius of curvature greater than 400 times the displacement thickness of the boundary layer the effect of curvature was negligible. On a concave surface the effect is relatively large; Re_t decreasing from 2 million to about 0.2 million as the radius of curvature decreases from infinity for the flat plate to 400 times the displacement thickness.

Effect of Roughness

The effect of a single cylindrical roughness element (a wire) on transition on a flat plate has been determined by many investigators and the results have been analyzed elsewhere [11] by the author. As the height is increased from a small value, the position of transition moves slowly forward from the smooth plate position until finally it reaches the position of the roughness element and remains there. Thus the roughness element produces a change in the transition location far downstream, Fig. 2 taken directly from Reference [11] shows that Re_t correlates very well with the value of the ratio of the height of the roughness element k to the displacement thickness δ_k^* of the boundary layer at the location of the roughness element, so long as transition is downstream and not too close to the roughness element. It also appeared from the data then available that the ratio of Re_t for the rough plate to Re_0 for the smooth plate was a single function of k/δ_k^* independently of airstream turbulence. However recent measurements by Tani, Iuchi, and Yamamoto [12] cast doubt on the general applicability of this relationship. Moreover in another paper [13] Tani has repeated the analysis of [11] using the original data recently found in Japan. There are numerous minor changes and one major one, namely Re_t for the smooth plate is approximately 1.7 million instead of 2 million assumed in [11]. Fig. 3 is a plot of Re_t/Re_0 vs k/δ_k^* from the corrected data.

The effect of three-dimensional roughness elements on transition seems entirely different. There is no effect until the element generates vorticity which spreads in a wedge behind the element. At my suggestion, G. B. Schubauer and P. S. Klebanoff of the National Bureau of Standards made some measurements with rows of spheres. The spheres have very little effect up to values of k/δ_k^* of 0.8 or more, the exact value depending on the size and location of the spheres. Above this critical height, transition moves very quickly to the sphere position. The behavior corresponds more nearly to a fixed critical Reynolds number of the sphere. Fig. 4 shows Re_t vs k/δ_k^* while Fig. 5 shows the same results plotted against $\frac{u_k k}{\nu}$ where u_k is the velocity in the boundary layer at a height k. The critical value of $\frac{u_k k}{\nu}$ varies from 490 to 760 with an average value of about 580. I greatly appreciate the courtesy of Schubauer and Klebanoff in permitting the use of these unpublished data. The effect of distributed random surface roughness on transition has not been studied very systematically.

Effect of Noise

When the surface is very smooth and the free-stream turbulence is less than 0.1 percent, transition may be induced by sound waves, either pure tones or random noise. This effect is described in [6] where it is shown that the boundary layer is more sensitive to a definite frequency band related to the value of R_{δ^*}, the boundary layer Reynolds number.

Effect of Surface Temperature

Liepmann and Fila [14] made some measurements of the effect of heating the surface of a flat plate on the location of transition. The value of Re_t was reduced from 0.5 million to 0.25 million for a surface temperature of $100^\circ C$. The heating produced velocity profiles in the boundary layer with inflection points.

Transition on Airfoils and Bodies of Revolution

We have reviewed the experimental data on the effects of stream turbulence, pressure gradient, curvature, roughness, noise and surface temperature on transition on a flat plate. The information on the effects of combinations of these five variables is very limited and much additional work would be required to fill in the gaps. But even if complete data were available, it is still not obvious how to apply the data to an airfoil or body of revolution in which effects of all the factors may be simultaneously present.

It is perhaps obvious that transition is a function of the local state of the boundary layer at the transition point, but it is not clear that the local state can be adequately described by mean velocity distribution, boundary layer Reynolds number, local pressure gradient, local free-stream turbulence, local surface curvature, and local roughness. We know that upstream roughness elements are important although their effect may be describable in terms of velocity fluctuations in the boundary layer at the transition point. But there is no prima facie reason for excluding a possible influence of higher derivatives of the pressure or other elements of the previous history of the boundary layer.

However many engineers hope that the transition Reynolds number based on local free-stream velocity and local displacement thickness of the boundary layer will exhibit relationships which permit transfer of experience from one body shape to another. Much more research must be done before such an approach can be adequately tested.

It is well known that spheres become relatively insensitive to free-stream turbulence when the turbulence level falls below 0.2 percent. This may be due to some detail of the pressure distribution around a sphere or it may mean that the blunt nose shape of the sphere produces disturbances which are greater than the free-stream turbulence.

Intermittent Nature of Transition

Although from measurements of mean speed, transition appears to be a gradual process extending over a transition region it has been known since [5] that transition in a boundary layer occurs suddenly just as Reynolds observed in a pipe with his dye filaments. Near the upstream limit of the transition region the rapid random fluctuations of speed characteristic of turbulence break in suddenly at random and at infrequent intervals. These turbulent bursts become more frequent and longer as the test probe is moved downstream until at the downstream limit the flow is always turbulent. These observations were interpreted as due to a statistical wandering of the transition point back and forth along the plate.

In 1951 Emmons [15] proposed that the turbulence originated locally in spots which then grew rapidly as they were carried downstream. This suggestion was based on experimental observation of such turbulent spots in a thin sheet of water flowing down an inclined plane. Very recently Mitchner, one of Emmon's associates, developed a technique [16] of initiating local disturbances in a boundary layer in air by a spark discharge through the layer. These artificial disturbances originate turbulent spots which grow as they move downstream. The growth of the spots can be studied by hot-wire measurements. Through the courtesy of Schubauer and Klebanoff of the National Bureau of Standards there is presented in Fig. 6 measurements of the growth of such a disturbance, its shape, and its characteristic oscillograph signature when it sweeps over a hot-wire probe. For comparison a record of natural transition is shown with unmistakably the same signature. There seems little doubt that transition to turbulence originates as Emmons suggested at many localized regions which grow in size as they travel downstream until they merge. The observed intermittency is due to the passage of such spots past the probe.

The Tollmien-Schlichting Theory of the Breakdown of Laminar Flow

The earliest theoretical attempts to account for transition from laminar to turbulent flow began with Reynolds himself who discussed the conditions under which a disturbance could absorb energy from the mean flow at a rate greater than the rate of viscous dissipation. The study of the stability of laminar boundary layer flow received the attention of the Göttingen group under Prandtl, of whom Tollmien and Schlichting were most active on this problem. A small two-dimensional disturbance is superposed on the basic two-dimensional boundary layer flow and the combined flow required to satisfy the Navier-Stokes equation. There results a linear homogeneous equation for the disturbance stream function. Its behavior is then investigated by studying the time history of a single Fourier component of selected frequency and wave length. This leads to a characteristic-value problem -- that is the boundary conditions are sufficient to determine the combinations of frequency, wave velocity, wave length, and damping which satisfy the disturbance equation for various boundary layer Reynolds numbers.

The solution shows that if the Reynolds number Re_{δ^*} exceeds a certain critical value, sinusoidal disturbances whose frequencies, wave velocities, and wave lengths lie within a certain band, are amplified. The theory, being based on small disturbances and a linearized equation can not follow the amplified disturbances until turbulence is generated.

From 1930 on an intensive experimental search was made for the Tollmien-Schlichting waves preceding transition. (For example see [5].) The search was finally successful in 1940 [6] when a wind tunnel airstream was available in which the free stream turbulence was less than 0.1 percent. Figs. 7, 8, and 9 show the frequencies, wave lengths, and wave velocities of the neutral oscillations which lie on the boundary between damped and amplified oscillations. When the existing disturbance is random and contains many frequencies, the selective amplification isolates a wave containing a narrow band of frequencies in the neighborhood of the frequency most highly amplified. This has been observed [6] in natural transition in a stream of low turbulence.

Stability of Flow vs Transition

It is now clear that instability of flow and transition are not identical. There are many other familiar examples in which the steady laminar flow is replaced by a flow periodic in time susceptible to theoretical treatment, for example the Kármán vortex street, the Taylor vortex cells between concentric rotating cylinders, etc. Such motions often are a prelude to turbulence but do not exhibit the random fluctuations characteristic of turbulence.

Effect of Finite Disturbances

When early experiments failed to disclose the Tollmien-Schlichting waves, attempts were made to study the behavior of boundary layers subjected to external disturbances simulating wind tunnel turbulence. In [5] a computation was made of the flow in a boundary layer for which the free-stream velocity varied sinusoidally with wave length λ and amplitude $\pm$ 2 percent about the mean velocity. Within 4 wave lengths separation occurred, its exact position depending on the phase of the wave relative to the leading edge of the plate. It was postulated that separation would give rise to turbulence. The computation showed an extraordinary sensitivity of the boundary layer to small pressure gradients.

A far more satisfactory computation was made later by Quick and Schröder [17]. Variation of the velocity with time as well as distance was assumed. A variation of $\frac{1}{2}$ percent of the mean velocity of wave length of the order of the boundary layer thickness gave separation within 3 wave lengths. The stream lines are shown in Fig. 10.

A more specific theory of the influence of turbulence on transition was developed by G. I. Taylor [18] based on the assumption that turbulence developed from momentary separation produced by the fluctuating pressure gradients accompanying the free-stream turbulence. According to this theory the transition Reynolds number was a function of $\frac{u'}{U}\left(\frac{l}{L}\right)^{1/5}$ when u'/U is the turbulence level, l is a reference dimension used in defining the Reynolds number and L is the scale of the turbulence. This was the commonly accepted theory of transition before 1940 and it received repeated experimental confirmation [19, 20]. In all cases however the free-stream turbulence was greater than 0.2 percent.

Speculative Picture of Transition in Incompressible Flow

Although much additional experimental and theoretical work is required to fill gaps and to establish convincing proof, it is possible to integrate the foregoing experimental data and theoretical computations into a reasonably clear picture of transition of the incompressible flow in a laminar boundary layer. In the general case the phenomenon proceeds in three successive steps: (1) the amplification of small disturbances; (2) the generation of localized areas or spots of turbulence; and (3) the growth and spread of the turbulent spots until the whole field of flow is turbulent.

The first step is found only when the disturbances are exceedingly small. Thus for free-stream turbulence less than 0.2 percent, surface roughness (except very near the leading edge) of height less than one-quarter the displacement thickness of the boundary layer, on bodies which show no separation of the flow from the surface, transition is preceded by the selective amplification of the small disturbances present.

The early history of this first stage is accurately described by the linear theory of Tollmien and Schlichting. Although the boundary layer is said to be unstable, this description is somewhat inaccurate. If a disturbance larger than those accidentally present is introduced into the flow and the boundary layer Reynolds number exceeds a critical value, selective amplification of a certain frequency band occurs; if the disturbance is removed, the amplified oscillations also disappear. The accuracy of the Tollmien-Schlichting theory has been established to the satisfaction of most aerodynamicists.

The initial disturbances which are present in any actual case may come from several sources. No situation seems conceivable in which all disturbances are absent because of the extreme sensitivity of the laminar boundary layer. For purposes of the theory the disturbances may be described by a composite spectrum of intensity vs frequency, although they actually arise from the turbulence of the free air stream, acoustic waves, surface roughness, and probably disturbances produced at the leading edge of the body. When small the effects of these disturbances are probably additive; when large the one contributing most energy in the critical frequency range probably governs, and this first stage in the transition process may be absent.

The wave lengths of the most highly amplified disturbances are long compared to the boundary layer thickness and vary with the Reynolds number. It is suggested that in many instances all sources of disturbance may be replaced by a spectrum of fluctuations of the free-stream velocity.

The turbulence of the free stream is usually isotropic and its spectrum is known in terms of the intensity and scale of the turbulence. It is important to note however that the spectrum is constant only in a statistical sense and the spectral analysis of the disturbances present at any one time may vary widely from the average spectrum.

Acoustic waves produce particle motions whose amplitude is comparable with turbulence intensities of a few hundredths of one percent. Effects of noise are observed only when all other disturbances are absent or extremely small. Its importance is probably greater in free flight than in wind tunnel experiments.

A single two-dimensional roughness element which is large enough to influence transition but small enough so that transition does not occur at the element may be regarded as generating a pulse, which contributes to the disturbance spectrum. It is well known that several such elements spaced at intervals of the critical wave length or an integral multiple contribute large amounts of energy in the critical part of the spectrum and produce early transition. Distributed roughness yields a random disturbance which can be expressed statistically in a spectrum in the same manner as free-stream turbulence.

As the free stream passes around the body it encounters a single pressure pulse of amplitude and shape determined by the pressure distribution. If the body is blunt, there may be sufficient energy in the critical wave length regions of its spectrum to be of importance. There may also be leading-edge disturbances associated with wandering of the stagnation point. The influence of body shape on transition is one of the least understood aspects. For bodies of revolution, the thickening of the boundary layer from continuity of flow requirements as the cross-sectional area diminishes at the rear may also initiate disturbances.

The first stage leaves us with laminar boundary layer oscillations of nearly sinusoidal frequency whose amplitude is increasing downstream. The second stage is the generation of local spots of turbulence. The mechanism of this stage is not yet fully understood. The process is surely connected with the non-linear terms in the Navier-Stokes equations. We have seen the rapidity with which local and momentary separation occurs when small disturbances are present. It is probable that the turbulence spots are produced in these regions of local separation. Theodorsen [21] has proposed one possible mechanism by which a localized region of slow moving air might generate vorticity. Görtler in recent conversations has proposed that the flow curvature obviously present in the disturbed laminar flow may generate three-dimensional vortices of the type observed on concave surfaces for which he has given the theory. Betz [22] gives some reasons for believing that vorticity is probably generated by the rolling up of vortex sheets. Such shear layers are of course present when separation occurs at a sufficiently low local Reynolds number. While the exact mechanism is still somewhat in doubt, we are reasonably sure that as the laminar boundary layer oscillations grow to large amplitudes, turbulence spots originate at various points in the flow field as suggested by Emmons [15].

The analogy with waves on the surface of water used by Emmons is helpful. If we observe the breaking of water waves in the open ocean as evidenced by white caps on the surface, we are observing the statistical effects of systems of waves which exhibit large amplitudes randomly distributed over the surface. As the amplitude of a water wave increases, a point is reached at which the effect of the non-linear terms in the governing equations produces a distortion of the sinusoidal shape and a folding over of the top. Under the force of gravity the top of the wave collapses. Similarly in the disturbed boundary layer, the statistical variations inherent in turbulence and random roughness produce maximum amplitudes in the field of flow which are randomly distributed and which vary in location from moment to moment. Each of these generates spots of turbulence.

When a single three-dimensional roughness element is of sufficient height, it generates vorticity behind it which spreads laterally in the familiar wedge of turbulence. This is a special case of a spot whose location is fixed. If there are a large number of such roughness elements, in a strip of emery paper for example, the wedges from the numerous elements meet in a short distance to produce transition just behind the roughess strip without the growth of random spots. Schubauer has found that the boundaries of the wedges of turbulence exhibit the intermittency characteristic of the outer edge of a turbulent boundary layer or wake, and that the laminar flow near the boundary exhibits large fluctuations.

When the free-stream turbulence is large, so little amplification is required to produce turbulence spots, that the stage described by the linear theory is unnoticed. "Large" in this sense is actually very small in absolute terms; laminar oscillations are unobservable in air streams in which the turbulence exceeds a few tenths of one percent.

The final stage in transition is the growth of the turbulent spots. Study of this process is in the very early stages but Schubauer and Klebanoff's measurements on artificially generated spots suggest an obvious mechanism. Their experiments were made in a laminar boundary layer on a plate in a region far forward of the natural transition point but above the minimum critical Reynolds number. Specifically for the case illustrated in Fig. 6 the minimum critical Reynolds number Re_{δ^*} is 450, the natural transition Reynolds number Re_{δ^*} about 2900 and the artifical spots were studied at an Re_{δ^*} of about 2100. A spot of turbulence is accompanied by large disturbances in the surrounding laminar flow. At Re_{δ^*} of 2100 a disturbance of 0.35 percent of the mean speed will produce transition. It is probable that the disturbance exceeds this amount and produces transition in the adjoining laminar fluid.

The shape of the spot in Fig. 6 is characteristic of a moving source which is propagating a flow disturbance in all directions at a finite speed less than the speed of the source. Thus like the finite speed of travel exhibited when one domino in a line of closely spaced dominoes standing on end is toppled against its neighbor, the breakdown of the laminar flow from the disturbance around a turbulent spot progresses at finite speed. If the speed of the spot is u_1 and the speed of travel of the breakdown of the flow is c, the vertex angle at the leading edge of the spot is $\sin^{-1} c/u_1$. For the case shown in Fig. 6 the leading edge moves at a speed very near the surface equal to 0.88 times the speed U_0 of the free stream and at a speed equal to the speed of the free stream at some distance above the surface, an average speed of the leading edge of $0.94\,U_0$. The trailing edge moves at a speed of $0.51\,U_0$. Hence the speed of the spot is $0.725\,U_0$ and the spot is expanding at a speed $0.215\,U_0$ from the center. The observed vertex angle at the leading edge is 15.3^o. Hence $c = 0.725\,U_0 \sin 15.3^o = 0.191\,U_0$. This speed is considerably less than the speed of the most highly amplified two-dimensional waves in the Tollmien-Schlichting theory, which lies in the range $0.27\,U_0$ to $0.35\,U_0$ at $R_* = 2100$. No theoretical calculations have been made of the most highly amplified three-dimensional disturbances in a laminar boundary layer but there is no apriori reason to expect it to be the same as either that for two-dimensional disturbances or that for the propagation of flow breakdown.

Whether this process is confirmed in detail remains to be seen.

If the description is correct, an artificially generated spot should not grow in a boundary layer for which Re_{δ^*} is less than 450 and the speed of flow breakdown may vary with Re_{δ^*}.

The speed of flow breakdown may also be computed from the half-angle θ of the turbulent wedge downstream from a single three-dimensional roughness element, since $c = U_0 \sin \theta$. For the case of Fig. 6 the observed half-angle of the turbulent wedge is 11.3°, giving $c = 0.196\, U_0$. The three values, 0.215 from rate of increase of spot length, 0.191 from the vertex angle at the leading edge, and 0.196 from the angle of the turbulent wedge are in fair agreement, the latter two being the more accurate. Had the leading edge speed of the spot near the surface been used in the first calculation, the three values would have been 0.185, 0.183, and 0.196.

Turbulent wedge angles observed under various conditions vary from 8.5° to 11° corresponding to values of c/U_0 from 0.148 to 0.190.

Mitchner [16] gave measurements on artificial turbulent spots in a laminar boundary layer at $R_{\delta^*} = 900$. The free-stream velocity was 550 inches/sec., the velocity of the leading edge of the spot was 450 inches/sec., and the velocity of the downstream edge was 310 inches/sec. From the rate of increase of the spot length c/U_0 is 70/550 or 0.127. The half angle of the turbulent wedge from the source was 8.6° giving c/U_0 equal to 0.150.

To recapitulate, transition appears to be the result of the sensitivity of the laminar boundary layer to disturbances lying within certain frequencies and wave lengths. Disturbances present inthis spectral region are amplified until local spots of turbulence appear, probably in regions of local and intermittent separation. Turbulence once generated, even in a very small region,itself produces disturbances of sufficiently large amplitude to spread the instability at a finite velocity which is less than the free-stream velocity.

Part II. Compressible Boundary Layer Flow

Additional Factors Affecting Transition in Compressible Flow

We shall now review the much more limited information available on transition in compressible boundary layer flow. The observed values of the x - Reynolds number range from about 0.5 million to about 28 million. The same factors that influence transition in incompressible flow also influence transition in compressible flow. Surface temperature and heat transfer however become of increasing importance and even when the body is thermally insulated the indirect effect of aerodynamic heating appears as an effect of Mach number. The surface temperature of the insulated body is considerably above the temperature of the free stream and the changes in density and velocity distribution accompanying the temperature changes modify the stability of the boundary layer.

A new factor at supersonic speed is the influence of shock waves.

When a weak shock wave strikes a laminar boundary layer, a small disturbance is produced analogous to the effect of a small roughness element. Stronger shock waves may either produce transition directly or produce laminar separation followed by transition in the free-shear layer and reattachment.

Effect of Free-Stream Turbulence

There are as yet few systematic experiments on the influence of free-stream turbulence on transition at supersonic speeds. There are few, if any, really low-turbulence supersonic wind tunnels and the technique and interpretation of hot-wire measurements in supersonic streams are in the early stages of development. The evidence is overwhelming however that the effects of turbulence are fully as great at supersonic speeds as at low speeds.

The flat plate at zero pressure gradient is not a very satisfactory test object in the available supersonic wind tunnels but a few scattered measurements have been made. An incidental measurement [23] in the NACA Ames 6-inch heat transfer wind tunnel at a Mach number of 2.4 gave Re_t based on free-stream properties of 1.4 million. The authors interpret the data as showing values as low as 560,000 at the lowest stagnation pressure. In another study [24] a crude analysis of skin friction data gave extrapolated values from 2.6 to 4.4 million. Measurements in the JPL 20-inch supersonic wind tunnel [25] gave values decreasing from 2.25 million at a Mach number of 2 to about 1.06 million at M = 3.6 increasing to 1.2 million at M = 4.5. A single measurement [26] in the GALCIT 5-inch hypersonic wind tunnel gave a value of at least 5 million at M = 5.8. Laufer [27] found that a change of 11 to 1 in turbulence level in the settling chamber of the JPL 20-inch tunnel did not measurably affect transition on a 5^{o} cone in the test section at Mach numbers above 2.5, although at M = 1.79 and a Reynolds number of 370,000 per inch, the increase in turbulence reduced Re_t from 4.5 million to 3.5 million. At M = 2.55, Re_t was about 3 million; at M = 4.5, Re_t was about 2.8 million.

The values of Re_t for cones and for bodies of revolution are not directly comparable with each other or with values for a flat plate. In few cases are the data available to compute $Re_{\delta *}$ and for $Re_{\delta *}$ an equivalent flat plate value of Re_t. Sternberg [34] notes that the thickness of the boundary layer on a cone is $1/\sqrt{3}$ times that on a plate at the same x - Reynolds number and therefore multiplies the value of 90 million obtained on a conical nose on the V-2 rocket at M = 27 by 0.58 to obtain an equivalent flat plate R_x of 52 million. In this paper the value of Re_t for the compressible flow data is the observed value without reduction.

Measurements of transition on a cone of 10^{o} total vertex angle have been made in numerous NACA wind tunnels [28]. The values vary from 400,000 to 7 million with no clear pattern of variation with Mach number or Reynolds number per foot. Turbulence measurements were not available. The design of the 1- x 1-foot variable Reynolds number wind tunnel at the NACA Lewis Laboratory has been twice modified to reduce its turbulence level, raising the maximum value of Re_t for the 10° cone from 700,000 to 1.3 million and then to about 4 million.

Another useful test object in supersonic wind tunnels is a hollow cylinder with sharp leading edge. Brinich [29] used such a hollow cylinder 5.31 inches outside diameter, 4.75 inches inside diameter, 33 inches long, with 5° beveled leading edge and 0.003-inch leading edge radius in the Lewis 1- x 1-foot variable Reynolds number wind tunnel. The Mach number was held constant and the Reynolds number varied by varying the pressure. Re_t varied with the pressure from 1.5 to 4 million increasing as the pressure increased. Brinich attributes this to the variation of the Taylor turbulence parameter with pressure at a fixed turbulence level. The physical picture is that the fluctuating pressure gradients are dependent on the ratio of the scale of the turbulence to the boundary layer thickness. As the density increases the boundary layer thickness decreases. Thus the scale of the turbulence is larger relative to the boundary layer thickness and has less effect in reducing the transition Reynolds number. On this view there would be both a Mach number and a density effect since the thickness increases with increasing Mach number, but Brinich's experiments were made at constant Mach number. In a wind tunnel of constant stagnation pressure increasing Mach numbers are accompanied by reduced density so that both effects would be in the direction to reduce transition Reynolds numbers with increasing Mach number for a fixed turbulence level when turbulence effects predominate in determining the location of transition. This effect is shown in the experiments of Lange, Gieseler and Lee [30] in the NOL 40 cm wind tunnel. The value of Re_t for a 5^o cone decreased from 3.4 million at M = 1.9 to one million at M = 4.2. For a hollow cylinder Re_t decreases from about 3 million at M = 2.3 to one million at M = 5. However the experimental picture is complicated by variations of the turbulence level with Mach number, stagnation pressure, compressor staging, etc.

Effects of Pressure Gradient, Curvature, Nose Shape

There are no data on the effects of pressure gradient, curvature, and nose shape on two-dimensional boundary layers except for measurements on airfoils, mainly at high subsonic speed. Since the pressure distribution varies greatly with the speed, these contribute little to an analysis of the basic problem.

There are a few investigations on bodies of revolution. Czarnecki and Sinclair [31] determined the transition Reynolds number for the RM-10, a body of revolution often used for comparative measurements in supersonic wind tunnels. The measurements were made in the NACA Langley 4-foot wind tunnel. The transition Reynolds number for a cone in this wind tunnel was about 6.5 million. The observed value for the RM - 10 was 11.5 million. A further investigation [32] was made in the same wind tunnel on two other bodies for comparison with the RM - 10, namely an ogive-cylinder and a cone-cylinder. All had a fineness ratio of 12.2 and measurements of transition and skin friction were made at Mach numbers from 1.2 to 2.2 and stagnation pressures from 1/8 to 2 1/4 atmospheres. The approximate transition Reynolds numbers were 2.5 million for the cone-cylinder, 4.7 million for the ogive-cylinder, and 11 million for the RM - 10.

Free-flight measurements by Jedlicka, Wilkins, and Seiff [33] in a pressurized range at the NACA Ames Laboratory gave a transition Reynolds number exceeding 12 million for a cone-cylinder model at a Mach number of 3.5. A much higher value (about 90 million) has been obtained on a V-2 rocket with a special conical nose [34] under transient conditions in the presence of heat transfer effects as discussed in a later section.

Effect of Roughness

A systematic study of the effects of single roughness elements has been made by Brinich [29] at a Mach number of 3.12. The basic model was a hollow cylinder as previously described and the roughness elements were wires encircling the cylinder. The diameter and location of the element were varied. The results were analyzed along the lines which had been effective in incompressible flow, the ratio of the transition Reynolds number for the rough plate to that for the smooth plate being plotted against the ratio of the height of the element to the displacement thickness of the boundary layer. The displacement thickness was that computed by the compressible flow formula. The scatter band was very wide but it was clear that compressible laminar boundary layers are much less sensitive to roughness than incompressible ones.

As previously mentioned the transition Reynolds number of the smooth plate varied with the stagnation pressure. Brinich compared the rough plate value with the smooth plate value at the same pressure, which appears the natural basis for comparison. However, if the comparison is made instead with the transition Reynolds numbers for the same transition position, the scatter is greatly reduced. Fig. 11 shows the recomputed points omitting three sets of data for elements closer to the leading edge of the plate than 25 k and omitting the rising leg of the curves corresponding to transition occurring at the element itself, as discussed in [11]. The correlation is good but there are noticeable systematic effects of x_k/k in the detailed results. The anomalous effect of the ratio increasing with increasing roughness height appearing in Brinich's Fig. 12 also disappears in the recomputed data.

The data for compressible flow are displaced to values of k/δ^*_k

about 3 times those for incompressible flow. This ratio is of the order of magnitude of the ratio of the free-stream density to that of the air striking the roughness element. One might speculate that the smaller effect is associated with this lower density of the air actually striking the element.

Some effects of distributed roughness on transition at a Mach number of 3.5 are described in [31].

Some observations on roughness effects at a Mach number of 5.8 are discussed in [26]. The boundary layer was markedly insensitive to roughness.

Effect of Heat Transfer

A body placed in a supersonic stream is heated above the surrounding air to a temperature different from that of the free stream. If the body is insulated and there is no transfer of heat by radiation, the body in steady flow will reach a constant temperature called the recovery temperature equal to the temperature of the air stream adjacent to the surface of the body. There will then be no heat loss by convection. The recovery temperature is somewhat less than the adiabatic stagnation temperature by an amount which depends principally on whether the boundary layer flow is laminar or turbulent. The values hitherto quoted for transition Reynolds number refer to insulated bodies without heat transfer although the surface temperature of the body increases with Mach number.

Heat transfer to or from a body exerts a marked influence on the transition Reynolds number, a flow of heat from the air stream to the body increasing the transition Reynolds number, and a heat flow from the body to the airstream decreasing it.

Higgins and Pappas [35] measured the effect of heating a flat plate at a Mach number of 2.4 and free-stream temperature of -205° F. to surface temperatures as much as 200° F. above the adiabatic recovery temperature of 60° F. The experiments were made in the NACA Ames 6-inch heat transfer tunnel in which transition occurred at Re_t equal to 1.25 million on the unheated plate. Heating to a surface temperature 200°F. above the recovery temperature reduced Re_t to 0.6 million.

The effect of heat transfer on transition on a 10° cone was determined by Scherrer [36] in the NACA Ames 1- by 3-foot supersonic wind tunnel No. 1. Transition on the unheated smooth cone at a Mach number of 1.99 occurred at Re_t equal to about 4 million. Increasing the surface temperature by 70° F. reduced Re_t to 3 million. To study the effect of cooling the cone was roughened by three grooves 0.01 inch wide and 0.015 inch deep to reduce Re_t for the unheated cone to about 2 million. Cooling the surface by 35° F. increased Re_t to about 3.3 million at $M = 2.02$. Cooling the surface by 58° at $M = 1.50$ increased Re_t to about 3.9 million.

Eber [37] describes measurements of transition on a cone-cylinder model, the cone having a total angle of 40°, as affected by heat transfer at a Mach number of 2.87. The results are complicated by the effects of the cone cylinder juncture. The unheated model showed a value of Re_t based on properties of the fluid at the model surface temperature of about 300,000. Raising the surface temperature about 125° F. reduces Re_t to about 160,000. Cooling the surface by 50° F. increases Re_t to about 450,000.

Czarnecki and Sinclair [31] made extensive measurements of the effect of heat transfer on a parabolic body of revolution (RM-10) at a Mach number of 1.61. The value of Re_t for the insulated body in the NACA Langley 4-foot tunnel was 11.5 million. Heating or cooling the body produced large changes in Re_t. The effects were determined for transition occurring always at the same location on the body, namely at the base of the body. The results are shown in Fig. 12 along with those from [35], and [37], Re_t being plotted against the ratio of the temperature difference between body and recovery temperature to the absolute stagnation temperature. Heating to a temperature-difference ratio of 0.3 reduced Re_t from 11.5 to 3 million whereas cooling to a temperature difference ratio of 0.15 increased Re_t from 11.5 to 28.5 million. The data from other sources show that the sensitivity to heat transfer decreases as the natural transition Reynolds number decreases. When the model surface was roughened, Czarnecki and Sinclair found a marked decrease in sensitivity to heat transfer effects. Cellophane tape on nose of the body at 3 and 25 percent of the body length reduced Re_t for the insulated body from 11.5 to 5 and 7.5 million. No increase in Re_t could be produced by cooling the roughened body.

Theory of the Stability of Compressible Flow in a Boundary Layer

The basic theoretical study of the stability of a compressible laminar boundary layer is that of Lees and Lin [38]. The authors concluded that the relation between wave length, wave velocity, and Reynolds number for neutral disturbances is of the same form as for incompressible flow. The solutions of the disturbance equation with the viscous terms neglected were studied in some detail. Lees continued the study [39] of the complete disturbance equation. An approximate value of the minimum critical Reynolds number was derived and numerical computations made for a number of cases. Lees concluded that heating the body above the recovery temperature reduces the critical Reynolds number, whereas cooling the body below the recovery temperature increases the critical Reynolds number. It was estimated that the boundary layer could be completely stabilized by a sufficient rate of heat transfer from the fluid to the body. For M = 1.5 the value of $\frac{T_s - T_i}{T_o}$ must be less than 0.175, at M = 3 less than 0.355 and at M = 5 less than 0.88 for complete stabilization.

The accuracy of Lees' computations and the correctness of his conclusions as to stabilization by cooling were soon challenged, leading to extensive discussions in the Readers Forum of the Journal of Aeronautical Sciences.

Bloom [40] repeated the stability computations and found that complete stability could be achieved by cooling only for Mach numbers less than 5.7 and that the amount of required cooling increases sharply for Mach numbers above 15. Lees' results were believed in error because of numerical errors and the incorrect assumption that a certain parameter is small.

At about the same time van Driest [41,] [42] studied the stability problem with somewhat different assumptions, using more accurate values of Prandtl number and viscosity-temperature relations. He concluded that no amount of cooling would stabilize the boundary layer above approximately Mach number 9.

In [43] Lees acknowledged numerical errors in [39]. Lin [44] confirmed that the assumption of small wave speed is not applicable to the high Mach number case but called attention to a more basic restriction on the results of the computations because of other approximations. Bloom [45,] [46] continued the discussion and outlined briefly his own revisions and some of the results to be published soon as an NACA Technical Note. His work raises further questions about the possibility of complete stabilization by cooling even for Mach numbers less than 5.7.

Weil [48] and Laurmann [49] have studied stability in the presence of a pressure gradient. Weil found the minimum critical Reynolds number greatly increased by a favorable pressure gradient at M = 1.5, becoming infinite at sufficiently high gradients. However at M = 4, the favorable effect became insignificant. Laurmann's results generally substantiate Weil's results.

Dunn and Lin [50], [51] have studied the relative stability of the compressible boundary layer to two- and three-dimensional disturbances. They show that for M = 1.6 and a wall-to-free-stream temperature ratio of 1.073, for which two-dimensional disturbances are completely stabilized, wavy three-dimensional disturbances propagated obliquely to the air stream yield critical Reynolds numbers of the order of one million.

It can be seen that the theory of the stability of a compressible boundary layer requires further study and numerical calculation. Nevertheless the qualitative behavior is not essentially different from that of the incompressible boundary layer. When the disturbances are small, the theory predicts a decrease in Re_t with increasing Mach number, but the accuracy of the theory at very high Mach numbers has been questioned.

Conclusion

A review of the current status of knowledge of the breakdown of laminar flow and the origin of turbulence in boundary layers shows substantial progress. The phenomenon of transition is not a capricious occurrence; it is dependent on many factors and often appears capricious because some of the factors are uncontrolled or unknown in many experimental measurements of transition. At subsonic speeds there are systematic experiments on the effects of free-stream turbulence, curvature of the surface, and single roughness elements, and a few measurements of the effect of pressure gradient, noise, surface temperature, and body shape. In general each of these factors has been studied singly with other factors constant at one or occasionally two levels. The number of possible combinations of the seven factors mentioned is extremely large and it is doubtful that any practicable experimental program could include all of them. We turn therefore to more fundamental and basic studies.

The fundamental studies have progressed rapidly and we seem to be approaching a break-through to a basic understanding of the origin of turbulence. The process of amplification of small disturbances is well understood with the emergence of localized high amplitude waves distributed randomly in time and space, random because the factors of free-stream turbulence and surface roughness are inherently random and because the instability of the boundary layer occurs for a band of wave lengths of varying wave velocity. Localized spots of turbulence originate in the regions of large amplitude by a mechanism not yet fully understood but in my opinion probably associated with local and intermittent separation. As first suggested by Emmons these spots grow in size as they move downstream. Detailed studies by Schubauer and Klebanoff follow-ing a technique of artificially generating turbulent spots first used by Mitchner are revealing the nature of this process. The spots are surrounded by regions of disturbance of the laminar flow of sufficient magnitude to break down the laminar flow in the adjacent fluid if the boundary layer Reynolds number is above the minimum critical Reynolds number. The break-down propagates at a finite rate until the whole flow is turbulent. The rate of propagation is that exhibited in the spread of turbulence behind an obstruction in the familiar wedge of turbulence, namely 0.15 to 0.20 times the speed of the free stream (wedge half-angle 8.5° to 11°).

Transition in compressible boundary layer flow appears to be controlled by the same factors as transition in incompressible flow with the addition at supersonic speeds of disturbances from shock waves impinging on the layer from sources of disturbance outside the boundary layer. The experimental data are less systematic except for the influence of heat transfer and single roughness elements. There seems little doubt that the phenomena are qualitatively similar.

No experimental studies of amplification of sinusoidal two-dimensional disturbances have been made in compressible flow, either at high subsonic or supersonic speeds, paralleling the studies of Schubauer and Skramstad at low speed. Neither theoretical or experimental studies have been made of the behavior of three-dimensional disturbances introduced into a laminar boundary layer either at subsonic or supersonic speeds. This would seem to be an especially fruitful field for further research.

It has been observed that transition at supersonic speed is intermittent. [33, 52] In a few of the schlieren photographs there is evidence of the presence of turbulent spots. As judged by the angles of turbulent wedges at supersonic speeds, the rate of propagation of flow breakdown is of the same order of magnitude at supersonic speeds as at low speeds, although at a Mach number of 5.8 [26] the observed value is lower, 0.10 the mean speed as compared to 0.15 to 0.20 at low speeds.

The net result of this summary is to emphasize the importance of the minimum critical Reynolds number. At any higher Reynolds number, the seeding of very small turbulent disturbances leads quickly to spreading the flow breakdown throughout the boundary layer. It is therefore important to clarify promptly the theory of stability of compressible boundary layer flow to determine whether in fact the minimum critical Reynolds number can be made infinite by cooling the body surface.

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FIGURE LEGENDS

Fig. 1. Effect of intensity of free-stream turbulence on the transition Reynolds number of a plate.

Fig. 2. Transition Reynolds Number and ratio of height of roughness element to boundary-layer displacement thickness at element from data of Tani, Hama, and Mituisi and Tani and Hama for two-dimensional roughness elements (wires). (From Reference 11).

Fig. 3. Ratio of transition Reynolds number of rough plate to that of smooth plate (for same transition position) and ratio of height of roughness element to boundary-layer displacement thickness at element, single cylindrical and flat strip elements (triangular symbols).

Fig. 4. Transition Reynolds Number and ratio of height of roughness element to boundary-layer displacement thickness at element for three-dimensional roughness elements (spheres) from unpublished data of G. B. Schubauer and P. S. Klebanoff.

Fig. 5. Transition Reynolds Number and Reynolds Number of roughness element for three-dimensional roughness elements (spheres) from unpublished data of G. B. Schubauer and P. S. Klebanoff.

Fig. 6. Growth of artificial turbulent spot in laminar boundary layer, from unpublished data of G. B. Schubauer and P. S. Klebanoff.

Fig. 7. Frequency of neutral oscillations excited in boundary layers by vibrating ribbon. (From Reference 6).
Solid curve is neutral curve according to Schlichting. Broken curves are neutral curves defined by experimental points.

Fig. 8. Wave length of neutral oscillations excited in boundary layer by vibrating ribbon. (From Reference 6).
Solid curve is neutral curve according to Schlichting. Broken curves are neutral curves defined by experimental points.

Fig. 9. Wave velocity of neutral oscillations excited in boundary layer by vibrating ribbon. (From Reference 6).
Solid curve is neutral curve according to Schlichting. Broken curves are neutral curves defined by experimental points.

Fig. 10. Streamlines for flow in the boundary layer of a plate subjected to periodically oscillating pressure variations beginning at distance L_o from the leading edge.
Amplitude of free-stream velocity variation is 1/2 per cent of mean value. Wave length L_w is 0.072 times initial length L_o.

Fig. 11. Ratio of transition Reynolds number of rough hollow cylinder to that of smooth hollow cylinder (for same transition position) and ratio of height of roughness element to boundary-layer displacement thickness at element, for single cylindrical roughness element from measurements of Brinich at a Mach number of 3.12.

Fig. 12. Effect of heating and cooling on transition Reynolds number.